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ON THE NAVIGATION OF AIRPLANES

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1. Introductory.

The present paper contains an account of investigations on the determination of the geographical positions of airplanes by means of sextant observations made during flight. These studies were made under the authority of the Division of Science and Research of the Bureau of Aircraft Production (in whose service the writer was at that time engaged as an engineer), and the present account is published by permission of Colonel Millikan, who was at that time in command of the Division. Most of the work was done at Langley Field, Hampton, Va., where every facility was afforded by the Commanding Officer, Major Howard, and those subordinate to him in authority. Observations in naval seaplanes were also secured through the courtesy of the Commanding Officer of the Naval Air Station at Norfolk. In the conduct of the observations, the writer received assistance of the greatest importance from Captain D. L. Webster (now Professor at the Massachusetts Institute of Technology), to whose skill in piloting a great part of the success of the investigation is due; from Mr. J. P. Ault, navigating officer of the non-magnetic vessel *Carnegie*, who was assigned to this work by the Department of Terrestrial Magnetism of the Carnegie Institution of Washington, and brought to it exceptional skill and experience in navigation at sea; and from Professor R. W. Willson of Harvard, whose bubble telescope proved to afford the best solution of the problem. Valuable aid and cordial interest in the work were also shown by many pilots and other officers at Langley Field and the Naval Air Station, to all of whom the writer offers his hearty thanks.

There is no difficulty whatever in handling a sextant in an airplane. It is desirable, tho not necessary, to use a rather small and light sextant, since the cockpit in which the observer sits is small, and the wind due to the motion of the "ship" is strong. For most

purposes it is better to remove the telescope altogether and observe with the naked eye, especially if the horizon is at all indistinct. If a telescope is used, it should be of low power and large field. Clearness of graduation and ease of reading of the arc are important, and a vernier that can be read with the naked eye is much better than one which requires a reading glass. It is often sufficient to read the arc alone, estimating tenths of a division on it in the ordinary fashion.

The continuous wind and the necessity of wearing gloves in winter, complicate the recording of observations. After some experiments, the writer settled upon the use of a small lap-board of heavy millboard, with the watch wired down at one corner, and a small, well-bound notebook held open upon it by two rubber bands. With a couple of pencils tied firmly to this board, the observer is no longer in danger of losing his records, and both sextant and notes may be held safely between his knees if the pilot enlivens the descent in an acrobatic fashion.

No difficulties of any account were met with in making observations at altitudes up to 16,000 feet, at speeds up to 105 miles per hour, or at the lowest temperatures which were encountered (about 20° Fahr.).

The principal difference between observations in the air and elsewhere consists in the greater variety of horizons which may be met with. Under favorable circumstances, the sea, or flat land, may afford an excellent natural horizon. At other times, the upper surface of clouds or of a layer of haze may serve as a horizon. Failing all these, some form of artificial horizon must be devised. The third of these alternatives is the most frequently encountered.

2. *Observations with Sea or Land Horizon.*

When the aircraft is above flat land or in sight of the sea and the air is clear enough, observations on the natural horizon can be made as easily as at sea, and with almost the same accuracy. Really level land, such as the coastal plain of Virginia, over which most of the observations were made, forms a better horizon than the sea, since it presents a greater contrast with the sky and can be seen from a greater altitude. The greatest height from which the sea horizon was observed was 10,600 feet; the land horizon, 12,400 feet—both during a short spell of exceptionally clear weather in August, 1918.

So long as the horizon can be seen at all, the accuracy of observation varies but little with the height of the airplane. This is illustrated by Table I, which includes all the results secured upon horizons of this type except those obtained during the first part of the observer's first flight.

TABLE I

Limits of Altitude (Feet)	Average Error of One Observation	Number of Observations
40 to 250	±2′.3	34
250 to 1,000	±2 .5	40
1,000 to 4,000	±3 .1	21
4,000 to 10,000	±3 .8	19
10,000 to 12,400	±3 .9	15
All Altitudes	±2 .9	129

A single observation of this accuracy suffices to fix the Sumner line with all the precision necessary in aerial navigation.

Among the usual corrections to the observed altitudes, the dip of the horizon may acquire remarkably large values. To test the proportionality of this to the square root of the height, the observations were reduced with the dip as the unknown quantity, and group means formed as given in Table II.

TABLE II.—DIP OF HORIZON

Mean Square Root of Height in Feet	Mean Observed Dip Minutes of Arc	Number of Observations
5	5	7
8	9	21
14	15	6
18	19	13
23	22	17
26	26	6
30	30	5
41	42	6
50	51	6
59	60	5
71	72	4
80	82	8
89	89	7
101	103	8
108	105	7

It is evident that the assumption that the dip in minutes is equal to the square root of the height in feet is substantially accurate at all heights. The ratio of the former to the latter, as determined from a least square solution from the tabular data, is 1.007 ± 0.004. As the altimeters used for reading the altitudes were not specially calibrated, great weight cannot be laid on the absolute value of this coefficient; but as the observations were made from three different

airplanes (and hence with different altimeters) during seven flights, including one at night, the mean value should be trustworthy. It indicates that the effect of refraction in diminishing the dip of the horizon is of about the same proportional amount at these great altitudes as at small heights. In deriving the tabular values, the refraction was applied with its full value for sea-level, but, as the observed altitudes were considerable, correction of the refraction to correspond with the lower density of the air at high altitudes would not affect the tabular data sensibly.

3. Observations on Cloud and Haze Horizons.

Under average conditions, the land or sea horizon is lost at an altitude of one or two thousand feet, since, even in "clear" weather, the lower air is usually full of haze. More often than not, however, this haze has a sharp upper boundary, at a definite level; and, when the ship has climbed well above it, it presents a definite "false horizon," which is always dark and usually sharp in the direction opposite to the Sun, but whitish and more diffuse under the Sun. Such a layer of haze may persist for weeks with variations in the height of its upper surface from day to day.

Observations upon such false horizons may be used provided that the height of the top of the haze above sea level is known, so that the dip may be computed. When, as often happens, the top of the haze coincides with the summits of low-lying scattered clouds, its altitude may be accurately determined when passing thru it on the ascent and descent. In the absence of clouds, the observer's estimate may be a few hundred feet in error.

A summary of the results obtained upon such horizons is contained in Table III. The first column gives the date; the second, the number of observations on the false horizon; the third, the average error, regardless of sign, of one observed altitude, corrected for the dip corresponding to the observer's height above the haze; the fourth, the error of the mean of all the observations, taking account of signs; and the fifth, the height of the top of the haze as estimated on the ascent and descent. The last three columns will be explained later.

On August 22nd, two horizons were simultaneously visible, one above the other; but only the observations on the lower one appear to be of value. With this exception, the observations in August and September are uniformly good enough to be useful.

The outstanding errors of the means may be explained by errors of a few hundred feet in estimating the height of the upper surface of the haze.

TABLE III.—OBSERVATIONS ON FALSE HORIZONS

Date	No. of Obs.	Average Error of one Obs.	Error of Mean	Observed Height of Haze (Feet)	Distance of Ridge (Miles)	Height of Ridge (Feet)	Average Residual Error
Aug. 16	5	±12′	+ 5′	6,600	....	.....	...
" 21	14	± 6	0	2,800	....	.....	...
" 23	26	± 6	+ 4	10,000	....	.....	...
" 23	5	±26	− 1	11,300	....	.....	...
" 24	12	± 4	+ 2	3,300	....	.....	...
Sept. 9	8	±11	−11	3,000	....	.....	...
Nov. 13	9	±22	−22	3,600	36	4,700	±8′
" 14	4	±26	−26	3,200	30	5,200	±4
" 25	7	±70	−70	3,200	31	7,000	±4
" 26	7	±69	−69	3,500	100	14,800	±2
" 29	19	± 7	− 7	4,000	....	.....	...
" 30	12	±18	+14	4,000	17.5	4,400	±5

On the contrary, the observations in November, when there was a persistent false horizon apparently similar to those seen in the summer, are quite valueless. This remarkable difference seems to indicate that the surface of the haze was in summer nearly uniform and level, while in late autumn it was irregular, so that the apparent boundary was not a true horizon but the top of a ridge of haze, rising above its general level. The dip of such a ridge below the theoretical horizon would vary linearly with the height of the observer, instead of being proportional to its square root; and, unlike a real horizon, it may be above the theoretical position and have a negative dip.

The observations of November 26th may be taken as an illustration. They are given individually in Table IV. The first column gives the observer's altitude; the second, the observed dip; the third, the dip computed for the haze surface at 3,500 feet which was passed thru and definitely located on both the ascent and descent; the fourth, the dip calculated from the empirical expression $0.0066\ (h-8200)$ where h is the observer's height in feet; and the last, the residual discordances between the observations and this formula.

The observed rate of change of dip, 6′.6 per thousand feet, corresponds to a ridge at the distance of 100 statute miles from the observer. In this distance, the Earth's surface curves away from the tangent by 6,600 feet; hence the top of the ridge, which lay on

the theoretical horizon of the observer at 8,200 feet, must have been 14,800 feet above sea level. The assumption that the observed "horizon" was actually such a ridge, represents the observations with an average residual of only $\pm 2'$, which is all that could be wished.

When it is further noticed that the observed horizon was in all cases far above the computed position of the haze horizon, and above the theoretical horizon for the last observation, there can be no doubt that the suggested explanation is the true one.

TABLE IV.—OBSERVATIONS ON CLOUD RIDGE

Height in Feet	Observed Dip	Computed Dip for Haze at 3,500 Feet	Empirical Formula	Residuals
11,800	+22′	+90′	+24′	−2′
10,900	+22	+87	+18	+4
10,300	+13	+82	+14	−1
9,800	+ 7	+79	+10	−3
9,300	+ 4	+76	+ 7	−3
8,900	+ 1	+74	+ 3	−2
8,200	0	+69	0	0
5,800	−15	+48	−16	+1

On this occasion it was noted at the time of observation that the "horizon" was apparently above the level of the false horizon to the northward, and was probably a strip of cloud. But at several other times observations on apparently good haze horizons gave similar results, which could in all cases be satisfactorily explained on the assumption of a distant ridge of haze. The computed height and distance of this ridge, derived in the manner just illustrated, are given in the sixth and seventh columns of Table III, and the average outstanding residuals in the last column. The latter are satisfactorily small in all cases. Except on November 30th, the observed horizon was higher than the computed position of the haze horizon, the ridge of haze concealing what lay beyond it. On the latter date it was lower, indicating that the surface of the haze beyond the ridge was lower than in the vicinity of the observer. On November 29th the observations were all made while the airplane was at nearly the same height, so that the character of the haze horizon could not be investigated.

On two days observations were secured above an almost continuous sea of clouds. When the observer was well above the clouds, their upper surface presented a beautifully sharp and definite horizon. The results obtained are summarized in Table V.

On the first of the two dates the upper surface of the clouds must have been very nearly level, as is shown by the excellent accordance of the observations. But on the second the circumstances were very different. The height of the upper surface of the clouds was carefully determined on the ascent, descent, and when flying home close over them. Yet the observations all make the Sun's altitude much too small. In the table they have been combined into groups, for each of which is given the mean height of the observer, the mean error of the observed altitude, and the average difference of one observation from these means. The residuals average only $\pm$ 1′.3, showing that the observations were very good.

TABLE V.—OBSERVATIONS ON CLOUD HORIZON

Date 1918	No. of Obs.	Height of Observer (Feet)	Height of Clouds (Feet)	Error of Mean Altitude	Average Deviation from Mean	Height of Clouds at Horizon (Feet)	Distance of Horizon (Miles)
Aug. 16	11	6,800 to 10,300	6,600	0′	±4′.5	....	..
Sept. 9	3	4,600	3,600	−30	±1.0	4,650	(3)
"	6	5,500	"	−22	±1.7	5,080	24
"	6	6,500	"	−18	±1.3	5,280	43
"	6	7,800	"	−15	±1.2	5,450	59
"	6	8,800	"	−12	±1.0	5,380	71
"	6	9,650	"	−12	±1.0	5,530	78
"	4	8,500	"	−14	±0.8	5,380	69
"	4	6,800	"	−13	±1.7	5,020	49
"	3	5,100	"	−16	±2.7	4,650	25

It is evident that the cloud horizon was considerably higher than its computed position, and the obvious explanation is that the upper surface of the clouds was actually higher in the region where the apparent horizon was situated than below the observer. As this region was out at sea, while the observer was well inland, the difference is not surprising. The last two columns of the table show the altitude of the cloud surface, which would explain the discordance for each group, and the corresponding distance of the horizon in statute miles. It is clear that the clouds rose steadily to seaward.

These observations do not cover a sufficient range of conditions to justify the drawing of general conclusions but it is evident that cloud or haze horizons, while sometimes good, are often unreliable and must be regarded with distrust.

It might be very inconvenient to have to descend to the level of the cloud or haze to find its height. This can be obviated by the

use of a dip-measuring instrument, which permits a direct measurement of the amount by which the angular distance of two opposite parts of the horizon differs from 180°. Observations with such an instrument were made by Mr. Ault on four flights. The error of a single determination of the dip was of the order of $\pm 3'$ to $\pm 5'$, and might have been diminished if the instrument had been specially adapted to the demands of aerial observation. By measuring the dip in this way, the observed altitude is effectively referred to the mean position of two opposite points of the horizon and errors of the sort discussed above should be much diminished.

4. Observations with Artificial Horizon.

Weather conditions under which the natural horizon is visible from any considerable altitude are unfortunately rare, and it is necessary to devise some substitute. This problem, which is equivalent to that of determining the true vertical in a moving airplane, is of much importance in aviation, but is difficult, since any bodily acceleration of the aircraft affects the apparent direction of gravity for an observer moving with it. The resulting deflection of the vertical may reach 45° in an ordinary steeply banked turn, and in a well made loop it suffices to reverse the apparent direction of gravity—the pilot and observer being held in their seats by centrifugal force, while head downwards.

Even small deviations from uniform flying will produce serious deflections. If a deviation of $15'$ is adopted as the greatest permissible, it is easy to show that the speed of the airplane must never be permitted to change at a rate exceeding one mile per hour in ten seconds, nor must the radius of curvature of the path be less than ten miles (assuming a speed of sixty miles per hour). This evidently demands very careful piloting, but, as will be shown below, such a standard can be attained under favorable conditions.

It might be supposed that this restriction could be escaped by the use of gyroscopic apparatus. But to maintain the *vertical*, any apparatus whatever must ultimately be controlled by gravity as a directive force, and an instrument which could discriminate between the components of the apparent gravity relative to the airplane which arise from the Earth's attraction and from the reaction due to acceleration of the motion would furnish an experimental disproof of the Principle of Generalized Relativity. (Incidentally, it would also be unaffected by the acceleration due to the rotation of the Earth, and therefore would not indicate the direction of the

vertical, as ordinarily defined, except at the equator or the poles.) All that can be hoped for from gyroscopic apparatus, therefore, is a lengthening of the free period of the swinging system, so that, when properly damped, it will not be sensibly affected by disturbing forces of short duration. To smooth out the irregularities in piloting, however, would require a free period of some minutes, and this would demand apparatus much too heavy and complicated for the present purpose. Damping against the rapid vibrations due to the engine is, of course, necessary, but can easily be provided.

The artificial horizon used in the present work was suggested by Major C. E. Mendenhall, and consisted of a pendulum mounted in gimbals so as to swing with equal freedom in any vertical plane, and bearing at its upper end a mirror adjustable so that its reflecting surface is horizontal when the pendulum swings freely. The lower end of the pendulum swings in a vessel of viscous liquid, which affords the necessary damping, while the whole instrument can, if desired, be supported on sponge rubber or other shock-absorbing material for further protection against vibration.

An experimental model was first constructed at the Science and Research Laboratory at Langley Field. The observations secured with this were so encouraging that two instruments of an improved type were constructed in the instrument shop of the Department of Terrestrial Magnetism, thru the courtesy of the Director, Dr. Bauer. These proved very satisfactory. The pendulum was about ten inches long, supported in ball-bearing gimbals, and carried a plane mirror of speculum metal, three inches in diameter. To protect the instrument from the strong winds due to the motion of the airplane, a cover with two glass plates inclined at about 40° to the horizon, and similar to those used on mercury horizons, was fitted over the top of the case.

The correct adjustment of the mirror is easily made by observing the double altitudes of some distant terrestrial object, and adjusting the levelling screws on which the mirror is mounted so that the same reading is obtained when the instrument is rotated about a vertical axis thru successive angles of 90°.

In observing with this instrument, double altitudes are measured just as in the case of a mercury horizon. During flight the image reflected by the mirror exhibits a small, rapid vibration due to the residual effects of the engine vibration and amounting usually to about ten minutes of arc. This makes it impracticable to set the

two images tangent to one another in the usual fashion, but the center of the apparently fixed "direct" image of the Sun can be set opposite to that of the rapidly vibrating reflected image with an accuracy abundantly sufficient for practical purposes.

When the two images are watched for a few moments, however, it is seen that the reflected image, in addition to its rapid vibration, exhibits a much slower and irregular oscillation in a vertical direction, arising from the deflections of the apparent vertical due to accelerations in the motion of the airplane. The magnitude of these oscillations depends upon the steadiness of the piloting, and they are not uniform either in period or amplitude. The image will often be stationary for a considerable fraction of a minute, then shift in a few seconds to a new position, sometimes a degree or more from the first, and retain this for many seconds, then shift quickly to still another position, and so on indefinitely. Under the best conditions, these oscillations do not seriously interfere with observation, except that to obtain the desired accuracy the mean of several observations must be taken. With less skilful piloting or in bumpy air the oscillations become large and good results can not be secured. The observer, after a little experience, can easily recognize the periods of steadiest flying, and observe during these. He should be provided with a "gunner's belt" in place of the ordinary safety belt, so that he may have as great freedom of motion as possible, and means should be provided for shifting the artificial horizon rapidly from one side of the "ship" to the other.

The results obtained with instruments of this sort are summarized in Tables VI and VII—the former giving the observations with the preliminary model, and the latter those with the final instrument. A few wildly discordant observations have been rejected mainly because notes made at the time showed that the ship may not have been flying in a straight line. The exclusion of such observations is obviously legitimate. No observations have been rejected except for reasons which would have been apparent to the observer while in the air, immediately after making them.

After the errors of the individual observations were computed, they were combined into groups in the order in which they were made, and the error of the mean of each group calculated. The number of observations in a group depends on the accuracy of the individual settings, being greatest when these are poor. As observations can be made and recorded at the rate of two a minute, it is

such a group, rather than the single setting, that should be regarded as the unit when estimating the practical value of the method. The first column in the tables gives the date; the second, the whole number of observations made with the artificial horizon; the third, the number rejected for cause, as explained above; the fourth, the average error, regardless of sign, of one of the remaining observations. The fifth column gives the number of groups into which the observations were combined; the sixth, the average number in a group (to the nearest unit); and the seventh, the average error, regardless of sign, for the mean result of one of these groups. At the foot of each table are given the mean values for all the observations made with each instrument, and for all the groups into which they were combined.

TABLE VI.—OBSERVATIONS WITH PRELIMINARY ARTIFICIAL HORIZON

Date 1918	No. of Obs.	Rejected	Average Error of One Obs.	No. of Group	No. in a Group	Average Error of One Group
Aug. 21	2	..	±26′	1	2	±26′
" 28	18	..	±23	2	9	± 7
" 29	35	4	±26	3	10	±13
" 31	17	..	±23	2	9	±17
Sept. 7	28	..	±26	3	9	±11
" 9	6	..	±30	1	6	±21
All these	106	4	±25	12	8.5	±15
Sept. 23	160	..	....	16	10	±14

The observations of September 23rd, were made by Mr. Ault.

TABLE VII.—OBSERVATIONS WITH FINAL ARTIFICIAL HORIZON

Date 1918	No. of Obs.	Rejected	Average Error of One Obs.	No. of Groups	No. in a Group	Average Error of One Group
Nov. 12	8	1	±25′	1	7	± 7′
" 13	35	..	±39	5	7	±14
" 14a	7	1	±25	1	6	± 1
" 14b	7	..	±33	1	7	±14
" 14b	(15)	..	(±47)	(2)	(7)	(±43)
" 15	9	..	±42	1	9	±25
" 18	12	..	±30	2	6	±22
" 20	21	..	±25	3	7	±21
" 27a	9	..	±31	1	9	±10
" 27b	19	..	±38	3	6	±14
" 27b	21	..	±12	4	5	±44
" 29	24	..	±12	5	4	± 4
Dec. 5	15	1	±17	3	5	±10
All	202	3	±30	32	6	±14.3
Rejecting observations on November 14						
All	187	3	±29	30	6	±12.4

The observations on the second flight of November 14th are recorded on two lines. The first refers to measures on the Moon, which was at a high altitude and well observable, while the second deals with observations on the Sun, which was very low, so that the observer's position while using the sextant was extremely cramped and uncomfortable. These observations, in the mean, place the Sun 43′ too high. In the cramped position of the observer, it was much easier to see the reflected image near the top of the oscillations arising from the accelerations of the airplane than at the bottom, so that it is very probable that the image was observed mainly at the top of the oscillations, which would account for the systematic error. A similar error, but smaller and of opposite sign, occurs in the observations of November 20th, and can also be explained by a cramped position of the observer in the narrow cockpit. It is clear that observations should not be attempted unless the observer's position is reasonably free, and it appears justifiable to exclude the worst of these two series from the general mean, as has been done in the last line of the table.

Tables VI and VII illustrate above all the importance of good piloting in observations with artificial horizon. The new instrument, which was mechanically far superior to the old, gave no more accurate results for the reason that the instrumental errors of both were negligible in comparison with the effects of accelerations due to the piloting. All the largest errors in Table VII are explicable in this way. For example, on November 13th the pilot stated after the flight that the control wires to the elevator were somewhat slack, and that it was therefore impossible to hold the air-speed as uniform as usual. As the Sun was observed over the tail, this accounts directly for the large accidental errors. The most interesting case is the second flight of November 27th. This was made in a large naval seaplane. The twenty-one observations summarized in the lower of the two lines assigned to this flight were made while the controls were in the hands of Lieutenant Barrin, a naval aviator of great experience, while during the nineteen previous observations the ship was handled by an army pilot (Captain Webster) thoroly experienced with military airplanes, but who had never flown a seaplane of this type before and had neither landmarks nor compass to steer by. The same pilot, handling ships of types familiar to him, secured the very accurate results of November 29th and December 5th.

It is evident that to secure the best results the controls must be in excellent adjustment, and that the pilot should be not only experienced but familiar with his ship. The last three lines of Table VII show what can be done under these circumstances. For fifty-nine observations, the average error of a single setting is $\pm 13'$, and that of a group of five observations is only $\pm 6'.4$. It is of interest to note that these three flights were made in ships of different types—HSIL (seaplane), De Haviland 4 and JN6H. In deriving these figures, a correction to the observations was made for the effect of wind upon the exposed mirror of the artificial horizon, which made the Sun appear $16'$ too high over the nose, and too low over the tail. The glass cover described above will eliminate this error.

5. Observations with Bubble Sextant.

The artificial horizon has several disadvantages: it is rather heavy, it requires to be shifted from side to side of the cockpit, and the observer's position is often inconvenient. These disadvantages are escaped by the use of an instrument devised by Professor R. W. Willson of Harvard University, who, upon learning of the present investigation, put it at the disposal of the observers, and came to Langley Field to help in the work.

Leaving a full description of this instrument to its inventor, it may be said here that it is an attachment to the telescope of an ordinary sextant, containing a level bubble moving freely on the under side of a spherical surface, and an unsilvered mirror inclined at $45°$ to the axis of the telescope, which reflects this bubble into the field of view. By an ingenious adaptation of the optical system, it results that a distant object which appears to be superposed on the center of the bubble will actually be on the true horizon, no matter at what point in the field of view both may appear to be.

In using the bubble telescope, it is attached to any ordinary sextant. An opaque screen is placed behind the horizon glass and the bubble is illuminated by the light of the sky thru the open end of the vertical bubble tube, or by a small electric lamp at night. The Sun is then "brought down" in the usual manner till it is visible along with the bubble in the field of view, and its image is then brought into the center of the bubble. The observations at first appear almost ludicrous—like trying to place an orange in the middle of a soap-bubble—but after a little practice readings may

be made very easily and with surprising precision. The average error of a single setting, when the observer is standing or sitting on a fixed support, is about 1'.5, and this might be diminished by using a smaller bubble. The one in the instrument sent by Professor Willson was apparently 2°15' in diameter.

The manipulation of the instrument is very similar to that of an ordinary sextant. A rolling clamp, devised by Professor Willson, which made it possible to move the index arm smoothly and rapidly thru any desired angle, large or small, proved a great convenience. An additional index error is introduced by any imperfection of adjustment of the unsilvered mirror; but this can easily be determined by observing the sea horizon with the bubble, or by taking a series of altitudes of the Sun with the bubble and with an ordinary artificial horizon (removing the screen behind the horizon glass in the latter case).

When using this instrument in the air the engine vibration practically disappears, the observer's body and arms forming a very efficient shock absorber, and the viscosity of the liquid surrounding the bubble supplying the damping. The wind usually offers no serious inconvenience. Accelerations of the motion of the airplane produce oscillations of the bubble, similar in character to those of the image reflected in the artificial horizon, but much less conspicuous, as they might be mistaken for the shifts of the bubble due to unsteadiness of the observer's hand, when he becomes tired. On the whole, observation is decidedly easier with the bubble sextant than with the artificial horizon, and it is a great advantage to have but a single self-contained instrument to work with.

Observations were successfully made with this sextant during flight upon the Sun and Moon by day, and the Moon and stars by night. The results are summarized in Table VIII, which is constructed similarly to Tables VI and VII.

TABLE VIII.—OBSERVATIONS WITH BUBBLE SEXTANT

Date 1918	No. of Obs.	Rejected	Average Error of One Obs.	No. of Groups	Obs. in Group	Average Error of One Group
Dec. 18a	6	..	±45'	1	6	±14'
" 18b	81	4	±30	8	10	± 7
" 19	35	2	±13	8	4	± 5
" 20	11	3	±12	2	4	±12
" 21	10	..	(±50)	..	..	....
1919						
Jan. 9	19	3	± 7	4	4	± 2
" 10	49	6	±13	6	7	± 7
" 11	54	4	±23	7	7	±12
" 15	13	1	±16	3	4	± 9
All but Dec. 21	268	23	±21	39	6.3	±7.4

Here again the results indicate the dependence of accuracy upon piloting. The very large errors on December 21st were obtained in a short flight with a pilot who apparently did not fully realize the nature of the problem. These results are therefore omitted from the mean. The first flight of December 18th was above clouds and the pilot had no landmarks to steer by. After the flight of January 11th, the pilot, who had been steering by compass over the sea, stated that the compass did not respond to small changes in the course until a perceptible time after they had occurred. This explains the relatively large errors on this flight. The second flight of December 18th is more interesting. This was a long trip, lasting nearly an hour and a half, and the average error of an observation increased steadily with the time. The mean results for groups of nine or ten observations in successive order were: $\pm 20'$, $\pm 30'$, $\pm 21'$, $\pm 30'$, $\pm 32'$, $\pm 30'$, $\pm 43'$ and $\pm 38'$. Upon showing these results to the pilot, Lieutenant Cleary, he remarked at once that toward the end of the flight he was getting tired and not flying as evenly as at the beginning.

The remaining five flights, which represent what can be done under favorable conditions, give an average error, regardless of sign, of $\pm 12'.8$ for one of the 112 individual observations, and of $\pm 6'.0$ for the mean of a group consisting on the average of 4.9 observations. On account of the high accuracy of these observations, the occasional wild readings due to momentary acceleration were more readily detected, whence the greater number of rejected observations. Captain Webster was the pilot on four of these five flights.

The bubble sextant appears to leave very little to be desired as an instrument for aerial navigation. It is light, portable, reasonably rugged, easy to learn to handle and to manipulate under practically all conditions, and permits of observations of a precision much surpassing the limits set by the residual accelerations in even the most careful piloting. Perhaps the best example of its capacities is found in the last flight, when nine observations were made upon *Sirius*, the observer being in the front cockpit, and having to turn in his seat and observe behind the wings, while the ship was carrying two bombs and consequently not flying easily. Nevertheless, the average error for one setting was $\pm 14'$, and for the mean of the nine $-2'.5$.

6. Reduction of the Observations.

If the ship's position is to be determined by calculations made in the air, these must be simplified and abbreviated as much as possible. St. Hilaire's method should evidently be used—computing the altitude and azimuth of the Sun (or other body observed) at some assumed geographical position for the time of observation and drawing the Sumner line on a chart at right angles to the Sun's azimuth, and at a distance from the assumed position corresponding to the "intercept" or difference between the observed and computed altitudes.

Probably the quickest way of doing this is to select an assumed position in advance[1] (which may be anywhere within one or two hundred miles of the position of the airplane) and to compute an ephemeris of the Sun's altitude and azimuth for equal intervals of apparent solar time at this point. Corrections for refraction and parallax (and even for the index error of the sextant, if this is constant) can then be applied to the altitude, so that the ephemeris to be taken into the air gives directly the apparent altitude which should be observed, if the airplane was at the assumed position. If the observing watch is set to apparent solar time at this point, all that is needed to determine the computed altitude corresponding to any observation is a simple interpolation in this table, and the intercept is found by subtracting this from the observed altitude (corrected for semidiameter and dip, if the natural horizon is used). If a chart has been prepared on which lines have been drawn from the assumed position in the direction of the Sun's azimuth at half hour intervals, the Sumner line may then be drawn immediately, using a draftsman's triangle, graduated in nautical miles on one of the perpendicular edges.

An ephemeris of altitude for a whole day can be calculated in less than half an hour if Ball's Tables[2] are used, and occupies a space no larger than a postcard, while the Azimuth Tables of the Hydrographic Office[3] simplify this part of the work equally. If the assumed position is taken on an even degree of latitude, only a single interpolation for the Sun's declination is necessary.

By such precomputation the work to be done in the air is reduced to a minimum, and the danger of numerical errors is also minimized. Even in a transatlantic flight, this method could be applied by

[1]Compare G. W. Littlehales, *U. S. Naval Institute Proceedings*, March, 1918, pp. 567-584.
[2]Published by J. D. Potter, London, 1907.
[3]U. S. Hydrographic Office, Pub. No. 71, 1918.

choosing in advance a series of standard positions, three hundred miles or so apart, along the track, and computing the altitude, etc., in advance for each, for such an interval of time as would safely cover the probable period when the ship would be near each position.

But such extensive precomputations are not imperative. All the calculation can be done in the air without serious loss of time. One quick method is to use Aquino's Tables, which, by a suitable choice of the assumed position (not usually on an even degree of latitude or longitude) reduce the calculations to a very simple form. If this is done, the azimuth must be computed and the Sumner line drawn separately for each observation; but nevertheless the method is thoroly practicable, as was proved by Mr. Ault during a flight from Langley Field to Washington and return. The average time required to secure a set of ten observations with the artificial horizon was 4.5 minutes, for the computations 3 minutes, and for the plotting of the Sumner line 2 minutes, with the aid of a special protractor.

Still better, probably, would be the calculating machine recently designed by Professor Charles Lane Poor[4], which resembles a circular slide rule. With this the altitude corresponding to any assigned latitude, time and declination can be found in one minute of time to within one or two minutes of arc, and the azimuth in half a minute more. The assumed position can be taken on an even degree to simplify the plotting. The present model of this instrument is a little large for convenient use in a small airplane but a smaller one could easily be made.

7. *Practical Results.*

The best evidence of the degree of success that has been obtained in actual practice is to be found in the results of three days' work when sights on the Sun and Moon could be obtained almost at the same time, and a "fix" obtained by the intersection of two Sumner lines. On the first of these days, the natural horizon could be used, and single sights sufficed to give good locations. On the other two the bubble sextant was used and the Sumner lines determined from the means of a number of successive sights.

The results are given in Table IX. The first column gives particulars concerning the flight, the horizon employed, and the difference between the azimuths of the Sun and Moon. The next

[4]See his book, *Simplified Navigation* (Century Co., New York, 1918). This calculating machine is now on the market, but is expensive.

three columns give the errors of the geographical positions determined from each set of observations, in latitude (positive if too far north), departure (positive if too far east), and in actual distance between the true and observed position—all in nautical miles. The last two columns give the number of observations on which each result is based. Each determination is wholly independent of the others.

TABLE IX.—POSITIONS BY SUMNER'S METHOD

General Data	Errors of Observed Position			Number of Observations	
	Latitude	Departure	Total	Sun	Moon
	+ 8	− 9	13	1	3
	− 2	− 5	5	1	1
Aug. 16, 1918; Over	− 1	− 3	3	1	1
York River; Air-	+ 4	− 7	8	1	1
plane, type JN4H;	+ 5	0	5	1	1
Natural Horizon;	− 3	− 6	7	1	2
Angle of Sumner	+ 2	− 6	6	2	1
Lines, 126°.	+ 1	− 9	9	1	1
	− 4	+ 2	4	1	1
	− 3	− 6	7	1	1
Jan. 10, 1919; Near					
Langley Field;	+ 2	+ 9	9	7	7
Airplane, JN4H;	+ 7	− 5	8	7	9
Bubble Sextant;	+13	+ 2	13	9	9
Angle 119°.					
Jan. 11, 1919; Off					
Cape Charles;					
Boat Seaplane,	+20	+14	24	8	8
type H-16; Bub-	+18	+11	21	8	8
bel Sextant; An-					
gle 141°.					

The average error of the actual location on the map for the ten determinations made with natural horizon is 6.7 nautical miles. The average time required to make the observations of one set was 2.3 minutes. For the five locations with the bubble sextant the average error is 15 miles, and the average time required for the observations, both of Sun and Moon, is 12 minutes. For the three sets on January 10th, when the piloting conditions were more favorable, the average error is only 10 miles.

So far as is known, these are the first occasions on which astronomical observations sufficient to determine completely the position of an airplane have been made during flight. They were not actually worked out in the air, as the object at that time was mainly to test the instrumental possibilities, especially as regards accuracy.

Mr. Ault, however, reports: "The first known instance of an airplane pilot being informed of his position by astronomical methods should be recorded here. During my flight from Langley Field to Washington, September 23, 1918, the visibility was very poor. The pilot, Lieutenant Charles Cleary, wished to verify his position, so slowed down and asked if the river below was the Potomac. I had just completed drawing in position line No. 5, which intersected our track at the Potomac river, so I was able to inform him that my observations placed us at the Potomac."

8. Observations of the Sun's Azimuth.

When only the Sun is observable but one Sumner line can be drawn. To find the observer's position on this line, Mr. Ault suggested that it might be possible to observe the Sun's magnetic azimuth, with a compass mounted in the airplane. Correcting for variation and deviation, the true azimuth is found, and this, under all ordinary circumstances, suffices to locate the point on the Sumner line at which the observation is made. From his report on preliminary investigations the following is abridged:

"A card graduated to half degrees from 0° to 360° was mounted on the preliminary artificial horizon mounting (so that its plane was horizontal) and a vertical pin mounted in the center of this card. The Sun's bearing was determined by noting the card reading of the shadow of the pin, reading simultaneously the magnetic heading of the airplane by the compass mounted nearby. The relation between the 0°-180° line of the card and the lubber line of the compass was determined when the airplane was "swung" on the ground to determine the deviations of the compass. On November 25, 1918, . . . nearly 120 observations of the Sun's azimuth were made on three different headings. The probable error of the mean of any ten observations was ±0°.6, and the mean difference between the deviations determined on the ground and those determined in the air was +0°.1. Only two of the twelve groups were in error over 1°." Similar observations made by Mr. Fleming, of the Department of Terrestrial Magnetism, gave the value ±0°.8 for the average discordance of the mean of a set of ten readings from a smooth curve of the theoretical form for the deviation of the compass. Such a set could be obtained in from one to one and a half minutes of observation.

Further studies of this obviously very promising method were interrupted by the necessary return of Messrs. Ault and Fleming to other duties. It is highly desirable that they should be continued, for they give good reason to hope that determinations of position, accurate enough to be of much value to aviators, can be made by observing the altitude and azimuth of the Sun, when it alone is visible.

9. Summary.

1. Sextant observations may be easily made from airplanes. More than a thousand sights on Sun, Moon and stars have been made during the present investigation from airplanes and seaplanes of nine different types.

2. When the natural horizon is visible a single sight gives the altitude with an average error of $3'$.

3. The relation that the dip of the horizon in minutes of arc is equal to the square root of the height in feet has been confirmed up to heights exceeding twelve thousand feet.

4. Horizons formed by clouds or haze, while sometimes valuable, are not reliable.

5. In the absence of a natural horizon, observations may be made with an artificial horizon or a bubble sextant. Both give good results, but the latter is lighter and much more convenient.

6. The chief error in such observations arises from the deviations of the apparent vertical, arising from irregularities of motion of the airplane. By very careful piloting, these have been reduced to an average of $\pm 13'$ for a single sight. With ordinary piloting they average twice as great and with careless piloting they would make observation useless.

7. It is therefore well to make several settings with the bubble sextant in rapid succession, and work up the mean of the group. The average error of the mean of groups of about six settings ranges from $\pm 12'$ for average piloting to $\pm 6'$ for very careful work. Such a group of observations can be obtained in from three to five minutes.

8. The necessary calculations may be much shortened either by doing most of the work before starting or by the use of rapid methods in the air. The whole time required for the plotting of a Sumner line from one sight or set of sights does not exceed five minutes, and may well be less.

9. When both the Sun and Moon were visible, determinations of position by Sumner's method showed an average error of location on the map amounting to 7 nautical miles when the natural horizon was available, and 15 miles with the bubble sextant (10 miles with the latter under good conditions). The whole time required for observation and calculation should be less than ten minutes in the first case and twenty minutes in the second.

10. Information regarding his position, determined with the sextant, has actually been furnished to a pilot during flight, and proved of use to him.

11. Methods of determining position from observations of the Sun alone, using its altitude and magnetic azimuth, have been provisionally tested with promising results.

It may fairly be concluded from the foregoing that the problem of finding the position of an airplane by astronomical means may be regarded as solved, within the limits set by the nature of the subject, and with sufficient accuracy for the practical purposes of aerial navigation. Over land, the pilot will doubtless be guided by landmarks, but on oversea flights and probably at times above clouds or fog, it is hoped that the investigations here recorded may prove of substantial usefulness.

Princeton University Observatory,
April 22, 1919.